ISSN: 0067-3064

DOI Link: https://doi.org/10.61586/dhFbf
Vol.38, Issue.7, Part.1, July 2025, PP.17-31

The numerical method for solution of the problem of two-dimensional (x-y) biphasic (water-crude oil) unsteady flow through porous media

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Received May 2025 Accepted June 2025 Published July 2025

Abstract:

This research presents a numerical model used to solve the problem of biphasic water-crude oil flow, under the influence of capillary pressure and the gravitational effect in a two-dimensional x-y space.
The novelty counts in the solution model of the system of equations resulting from the development in finite differences which led to the removal of some errors that affected especially the saturation values.
For the case of water-crude oil mixture flow from a core, the model was successfully tested and the results were compared with the analytical solution, which highlighted the advantages of using this method compared to others.

Keywords: final recovery, numerical solution, models, fundamental equations, pressure, water, crude oil.

ISSN: 0067-3064

1. Introduction

To increase the final recovery factor in crude oil reservoirs, tertiary recovery methods must be applied. The relatively high cost of tertiary recovery processes has imposed new requirements for the accurate determination of design parameters, thereby improving the forecasting methods for hydrocarbon reservoir exploitation [1].

The simplified (analytical) mathematical models used to model the flow in hydrocarbon reservoirs (oil and gas) are no longer up-to-date, they can only be applied for particular cases of the reservoir, they do no take into account the geometry of the flow, the inclination of the reservoir, the placement of wells and the influence their role in the dislocation process, the variable thickness of the reservoir, the non-uniformity of the parameters of the reservoir horizontally and vertically, the exploitation history.

These conditions led to the development of complex mathematical models that can no longer be solved analytically, necessitating the use of specialized software to simulate the exploitation process in the petroleum field.

The mathematical models offer a significant advantage to the oil extraction industry, as they enable the simulation of various exploitation processes, allowing for optimal decisions to be made regarding the exploitation of the reservoir.

The research presents a numerical model for two-phase water-crude oil flow under the influence of capillary pressure and gravitational effects, which can be used to design the exploitation of crude oil reservoirs with or without a history, by water injection or with partially or fully active aquifers. On the other hand, the thickness of the reservoir, as well as its inclination and geometry, can be variable, covering a wide range of reservoir types in our country [4].

The development of the model takes into account the non-uniform distribution of reservoir parameters in both time and space. On the other hand, specialized literature offers several numerical methods for solving such a problem. In the present research, the simultaneous solution method (SSM) was employed by implicitly considering *Pt* and *Pa* (and therefore their saturations) with explicit transmissibility.

Thus, iterative models can be used to solve the resulting algebraic system of equations (Jacobi, Gauss-Seidel). The best results were obtained using the simultaneous solution through coupled equations method (SSECM), an original technique developed at the Petroleum and Gas University of Ploiesti. This method can be applied to any of the iterative variants; in the present research, the Gauss-Seidel sub-variant was used.

Also, the development calculation algorithm was applied for testing in the case of linear flow in a tube (any results were compared with the analytical solution obtained by using the Buckley-Leverett theory) [2].

The results obtained demonstrated the reliability and viability of the developed model in reality.

2. The description of the method

a) Determination of the numerical model for the biphasic flow (water-crude oil) in the two-dimensional space under the influence of the capillary pressure and the gravitational effect

Consider a porous-permeable reservoir of porosity m, absolute permeability K, relative permeability for the water phase k_a and for the crude oil phase k_t, saturated with saturation water S_a and saturation crude oil S_t, in which there is a source of crude oil specific volume flow (for unit volume) q_t and a water supply with specific volume flow rate q_a.

The flow is considered incompressible so that the volume factors for the two phases (water-b_a and crude oil-b_t), specific masses (water-ρ_a and crude oil-ρ_t) and dynamic viscosities (water-μ_a and crude oil-μ_t) are constant in time and space [5].

To determine the mathematical law of biphasic flow and implicitly the flow equations, the three fundamental equations of underground hydraulics, written for each phase separately, are taken as a reference [3]:

- continuity equation;
- Darcy's equation;
- the equation of state.

In Cartesian coordinates of the two-dimensional space, the continuity equation is:

- for the water phase:

$$-\left[\frac{\partial}{\partial x}(\rho_a \cdot \vartheta_{ax}) + \frac{\partial}{\partial y}(\rho_a \cdot \vartheta_{ay})\right] + M_{sa} = \frac{\partial}{\partial t}\left(\rho_a \cdot m \cdot \frac{S_a}{b_a}\right) \quad (1)$$

- for the crude oil phase:

$$-\left[\frac{\partial}{\partial x}(\rho_t \cdot \vartheta_{tx}) + \frac{\partial}{\partial y}(\rho_t \cdot \vartheta_{ty})\right] + M_{st} = \frac{\partial}{\partial t}\left(\rho_t \cdot m \cdot \frac{S_t}{b_t}\right) \quad (2)$$

For the case of one-dimensional motion, Darcy's equation is:

- for the water phase:

$$\overrightarrow{\vartheta_a} = -\frac{K \cdot k_a}{\mu_a \cdot b_a} \cdot grad\Phi_a \quad (3)$$

- for the crude oil phase:

$$\overrightarrow{\vartheta_t} = -\frac{K \cdot k_t}{\mu_t \cdot b_t} \cdot grad\Phi_t \quad (4)$$

Where the phase potentials are defined:

$$\Phi_a = P_a - \rho_a \cdot g \cdot z \quad (5)$$

$$\Phi_t = P_t - \rho_t \cdot g \cdot z \quad (6)$$

where:

z – the elevation of the point relative to a horizontal reference surface (figure 1);

g – gravitational acceleration;

P_a and P_t – pressures in the water and crude oil phases, related by capillary pressure:

$$P_c = P_t - P_a \quad (7)$$

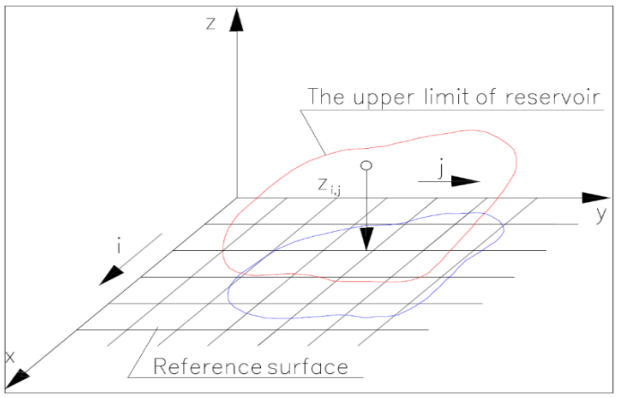

Figure 1 – The definition domain

In equations (2 and 3) we can express:

$$grad\Phi_f = \frac{\partial \Phi_f}{\partial x} \cdot \vec{i} + \frac{\partial \Phi_f}{\partial y} \cdot \vec{j}, f=a,t \quad (8)$$

Given incompressible flow, the equations of state have the form:

$$\rho_t = cont., f = a, t \quad (9)$$

Substituting equations (2and 3) and (9) into the continuity equations and making the simplifications, the equations of motion are obtained in the form of the system of differential equations:

$$\nabla\left(\frac{K \cdot k_a}{\mu_a} \cdot \nabla \cdot \Phi_a\right) + b_a \cdot q_a = m \cdot \frac{\partial S_a}{\partial t} \quad (10)$$

$$\nabla\left(\frac{K \cdot k_t}{\mu_t} \cdot \nabla \cdot \Phi_t\right) + b_t \cdot q_t = m \cdot \frac{\partial S_t}{\partial t} \quad (11)$$

where:

$$\nabla = \frac{\partial}{\partial x} \cdot \vec{i} + \frac{\partial}{\partial y} \cdot \vec{j} \quad (12)$$

The volumetric flow rates will be:

$$q_a = \frac{M_{sa}}{\rho_a} \quad (13)$$

$$q_t = \frac{M_{st}}{\rho_t} \quad (14)$$

According to specialized literature, saturations are related to the relationship:

$$S_a + S_t = 1 \quad (15)$$

Two-dimensional flow is an important problem with initial and boundary conditions applied in the closes domain. So that, at the boundary of the reservoir there is no flow, Neumann type boundary conditions (in derivative form) will be considered, and the initial condition will be considered the crude oil phase pressure, uniformly or non-uniformly distributed at the reference level z=0.

For the numerical solution of system (7) the boundary conditions require the use of a block-centered mesh (figure 2).

When writing the system (9) in finite differences, we take into account the fact that: $S_a=S_a(P_c)$, (figure 3) as:

$$P_c = \Phi_t - \Phi_a + (\rho_t - \rho_a) \cdot g \cdot z \qquad (16)$$

So that:

$$\frac{\partial S_a}{\partial t} = \frac{\partial S_a}{\partial p_c} \cdot \left(\frac{\partial \Phi_t}{\partial t} - \frac{\partial \Phi_a}{\partial t}\right) \qquad (17)$$

Expanding in finite differeces around node (i,j), the mesh step is (figure 4):

- Δx along the x direction;
- Δy along the y direction and after multiplying by the volume of a cell $\Delta x \Delta y H$, system (9) becomes:

$$\Delta T_a \Delta \Phi_a^{n+1} + Q_a \cdot b_a = G(\Phi_t^{n+1} - \Phi_t^n) - G(\Phi_a^{n+1} - \Phi_a^n) \qquad (18)$$

$$\Delta T_t \Delta \Phi_t^{n+1} + Q_t \cdot b_t = G(\Phi_t^{n+1} - \Phi_t^n) - G(\Phi_a^{n+1} - \Phi_a^n) \qquad (19)$$

Considering progressive finite difference, it can be write:

$$\frac{\partial \Phi_f}{\partial t} = \frac{\Phi_f^{n+1} - \Phi_f^n}{\Delta t}, f = a,t \qquad (20)$$

where:

Δt – the time step;

H – the thickness of the layer at the point where the development was done and where the volumetric flow rates were noted:

$$Q_a = q_a \cdot \Delta x \cdot \Delta y \cdot H \qquad (21)$$

$$Q_t = q_t \cdot \Delta x \cdot \Delta y \cdot H \qquad (22)$$

The flow rates are considered positive for injection and negative for extraction.
It was noted:

$$G = \frac{m \cdot \frac{\partial S_a}{\partial p_c} \Delta x \cdot \Delta y \cdot H}{\Delta t} \qquad (23)$$

The development operators in finite differences are f=a,t):

$$\Delta T_f \Delta \Phi_f = \Delta x T_{xf} \Delta x \Phi_f + \Delta y T_{yf} \Delta y \Phi_f \qquad (24)$$

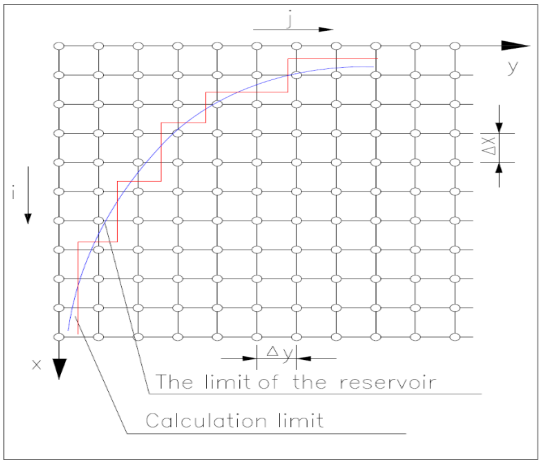

Figure 2 – Modeling of the reservoir boundary

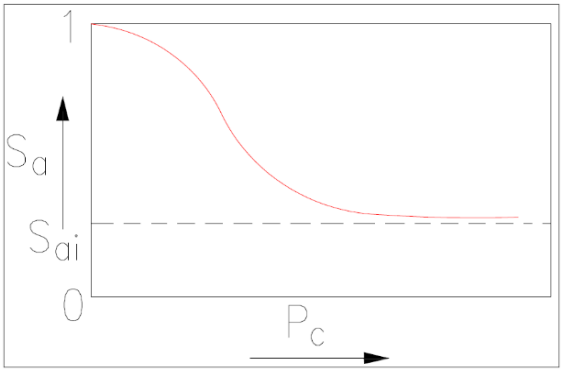

Figure 3 – The dependence $S_a = S_a (P_c)$

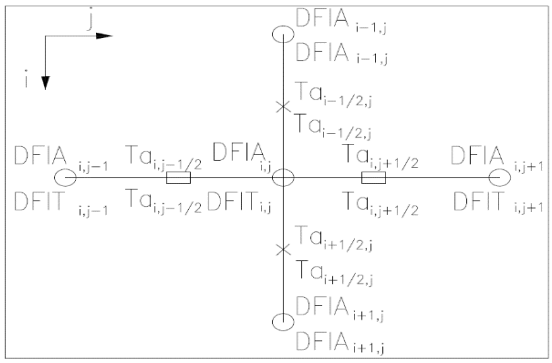

Figure 4 – Discretization network element

The upper index (n+1) indicated the time level:

$$t_{n+1} = t_n + \Delta t \tag{25}$$

and transmissibilities are:

$$T_{xf} = \frac{K \cdot k_f}{\mu_f} \cdot \frac{\Delta y}{\Delta x} \cdot H;\ T_{yf} = \frac{K \cdot k_f}{\mu_f} \cdot \frac{\Delta x}{\Delta y} \cdot H \tag{26}$$

To ensure the balance (10) in equation (22) will be written:

$$\frac{\partial S_a}{\partial p_c} = \frac{N-\nu}{N} \cdot S_a'^{\nu} + \frac{S_a^{\nu+1} - S_a^n}{P_c^{\nu+1} - P_c^n} \cdot \frac{\nu}{N} \tag{27}$$

where:

$S_a'^{\nu}$ is the slope of the curve $S_a(P_c)$ at iteration level ν of the total N iterations used to solve the algebraic system of equations resulting from the development in finite differences.

In order to ensure the stability of the numerical solution, the relative phase permeability within the transmissibilities will be evaluated upstream, thus [1]:

$$T_{f_{i+\frac{1}{2},j}} = \frac{\Delta x}{\Delta y} \cdot \frac{1}{\mu_f} \cdot \frac{1}{4} \cdot \left(H_{i,j} + H_{i+1,j}\right) \cdot \left(K_{i,j} + k_{i+1,j}\right) \cdot \left[\gamma \cdot k_{f_{i,j}} + (1-\gamma) \cdot k_{i+1,j}\right] \tag{28}$$

$$T_{f_{i,j+\frac{1}{2}}} = \frac{\Delta y}{\Delta x} \cdot \frac{1}{\mu_f} \cdot \frac{1}{4} \cdot \left(H_{i,j} + H_{ij+1}\right) \cdot \left(K_{i,j} + k_{i,j+1}\right) \cdot \left[\gamma \cdot k_{f_{i,j}} + (1-\gamma) \cdot k_{i,j+1}\right] \tag{29}$$

where:

$$\gamma = 0 \rightarrow \left(\Phi_{f_{i,j}} \leq \Phi_{f_{i+1,j}}\right) \text{ or } \left(\Phi_{f_{i,j}} \leq \Phi_{f_{i,j+1'}}\right) \quad (30)$$

$$\gamma = 1 \rightarrow \left(\Phi_{f_{i,j}} \geq \Phi_{f_{i+1,j}}\right) \text{ or } \left(\Phi_{f_{i,j}} \leq \Phi_{f_{i,j+1'}}\right) \quad (31)$$

In the nodes where the wells are located, the boundary conditions are determined, which can be: the pressures in the wells or the flow rates.

In the present research, the case where the flows are specified is considered.

If in equations (22) Q_t represents the extracted crude oil flow rate and Q_a (equation 23) represents either the extracted water flow rate through the extraction wells deduced from the well saturations and the extracted crude oil flow rate, or the injection flow rate for the nodal points where they are placed these probes, results:

$$\Delta T_a \Delta \Phi_a^{n+1} - G(\Phi_t^{n+1} - \Phi_t^n) + G(\Phi_a^{n+1} - \Phi_a^n) = -b_a Q_a \quad (32) \quad \Delta T_t \Delta \Phi_t^{n+1} + G(\Phi_t^{n+1} - \Phi_t^n) - G(\Phi_a^{n+1} - \Phi_a^n) = b_t \Phi_t \quad (33)$$

Adding and subtracting from each member of the system (32 and 33) the term $\Delta T_t \Delta \Phi_t^n$ *(f=a,t)* and noting the displacements of the potentials in each phase, we obtain:

$$DFIA^{n+1} = \Phi_a^{n+1} - \Phi_a^n \quad (34)$$

$$DFIT^{n+1} = \Phi_i^{n+1} - \Phi_7^n \quad (35)$$

The system (34 and 35) takes shape:

$$\Delta T_a \Delta DFIA^{n+1} - GDFIT^{n+1} + GDFIA^{n+1} = -\Delta T_a \Delta \Phi_a^n - b_a Q_a \quad (36)$$

$$\Delta T_f \Delta DFIT^{n+1} + GDFIT^{n+1} - GDFIA^{n+1} = -\Delta T_f \Delta \Phi_t^n + b_t Q_t \quad (37)$$

where Q_a represents the flow of water extracted through the production wells and can be written:

$$Q_a = -Q_t \frac{k_a}{k_t} \cdot \frac{\mu_t}{\mu_a} \quad (38)$$

Notting the residues from the system (38) it follows:

$$RA = \Delta T_a \Delta \Phi_a^n + b_a Q_a \quad (39)$$

$$RT = \Delta T_t \Delta \Phi_t^n - b_t Q_t \quad (40)$$

Developing for each node (i,j) the algebraic system generated by the equations can be obtained:

$$T^n_{a_{i-\frac{1}{2},j}} \cdot DFIA^{n+1}_{i-1,j} + T^n_{a_{i,j-\frac{1}{2}}} \cdot DFIA^{n+1}_{i,j-1} - \left(\sum T^n_{a_{i,j}} - G\right) \cdot \cdot DFIA^{n+1}_{i,j} + T^n_{a_{i+\frac{1}{2}j}} \cdot DFIA^{n+1}_{i,j} + T^n_{a_{i,j+\frac{1}{2}}} \cdot DFIA^{n+1}_{i,j+1} - -G \cdot DFIT^{n+1}_{i,j} = -RA \quad (41)$$

$$T^n_{t_{i-\frac{1}{2},j}} \cdot DFIT^{n+1}_{i-1,j} + T^n_{t_{i,j-\frac{1}{2}}} \cdot DFIT^{n+1}_{i,j-1} - \left(\sum T^n_{t_{i,j}} - G\right) \cdot \cdot DFIT^{n+1}_{i,j} + T^n_{t_{i+\frac{1}{2}j}} \cdot DFIT^{n+1}_{i+1,j} + T^n_{t_{i,j+\frac{1}{2}}} \cdot DFIT^{n+1}_{i,j+1} - -G \cdot DFIA^{n+1}_{i,j} = -RT \quad (42)$$

This system of equations can be solved by various usually iterative methods.

Given the fact that solving the system of equations results in pressures in each phase that are linked by means of the saturation capillary pressure, it means that the saturations are implicitly calculated.

For this reason this method is called the Simultaneous Solution Method (SS Method) [7].

The main iterative method for the SS Method (Gauss-Seidel Method) as well as the improved version of the SS Method called the Simultaneous Solution Method by Coupled Equations (SSEC Method) will be presented next.

b) Determination of the system of equations and simultaneous solution by the Gass-Seidel Method (SS Method)

The equations of the system (41 and 42) written in each node (i,j) of the discretization network, generate a system of equations with block matrices, in which the equations for each phase appear successively [6].

By applying the Gauss-Siedel Method for each of the system equations, the iterative recurrence relations used for the successive estimation of the unknowns $DFIA_{i,j}$ and $DFIT_{i,j}$ are obtained as follows:

$$DFIA_{i,j}^{\nu+1} = \left(RA + T_{a_{i-\frac{1}{2},j}}^{n} \cdot DFIA_{i-1,j}^{\nu+1} + T_{a_{i,j-\frac{1}{2}}}^{n} \cdot\cdot DFIA_{i,j-1}^{\nu+1} + T_{a_{i+\frac{1}{2},j}}^{n} \cdot DFIA_{i+1,j}^{\nu+1} + T_{a_{i,j+\frac{1}{2}}}^{n} \cdot DFIA_{i,j+1}^{\nu} - -GDFIT_{i,j}^{\nu}\right) / \left(\sum T_{a_{i,,j}}^{n} - G\right) \quad (43)$$

$$DFIT_{i,j}^{\nu+1} = \left(RT + T_{t_{i-\frac{1}{2},j}}^{n} \cdot DFIT_{i-1,j}^{\nu+1} + T_{t_{i,j-\frac{1}{2}}}^{n} \cdot\cdot DFIT_{i,j-1}^{\nu+1} + T_{t_{i+\frac{1}{2},j}}^{n} \cdot DFIT_{i+1,j}^{\nu+1} + T_{t_{i,j+\frac{1}{2}}}^{n} \cdot DFIT_{i,j+1}^{\nu} - -GDFIA_{i,j}^{\nu}\right) / \left(\sum T_{t_{i,,j}}^{n} - G\right) \quad (44)$$

where: υ represents a know literacy level.

The iterative process continues until the convergence conditions are met, and re-initialization can then be done.

3. Improvement of the simultaneous solution method by coupled equations (SSEC method)

In accordance with the results of the classical iterative method for solving systems generated by multiphase flows, each equation in the system is treated separately, not considering the interdependence of the phase pressures in each node.

This leads to a different propagation of errors in each equation which affects the saturation value calculated implicitly according to these pressures. Therefore, to increase the convergence rate, the simultaneous treatment of the two equations is required, considering them as forming a subsystem of the general system (9).

The equations of system (41,42) can be written in the form:

$$T_{i-\frac{1}{2},j}^{n} \cdot DFI_{i-1,j}^{n+1} + T_{i,j-\frac{1}{2}}^{n} \cdot DFI_{i,j-1}^{n+1} - S_{i,j}^{n} \cdot DFI_{i,j-1}^{n+1} + +T_{i+\frac{1}{2},j}^{n} \cdot DFI_{i+1,j}^{n+1} + T_{i,j+\frac{1}{2}}^{n} \cdot DFI_{i,j+1}^{n+1} = -R \quad (45)$$

where it was noted:

$$DFI_{i,j}^{n+1} = \left|\begin{matrix} DFIA \\ DFIT \end{matrix}\right|_{i,j}^{n+1}, S_{i,j}^{n} = \begin{vmatrix} \sum T_{a_{i,j}}^{n} - G & G \\ G & \sum T_{t_{i,j}}^{n} - G \end{vmatrix}, R = \left|\begin{matrix} RA \\ RT \end{matrix}\right| \quad (46)$$

To apply the Gauss-Seidel Method from the equation (41,42) for the iteration level υ write:

$$DFI_{i,j}^{\nu+1} = \left(R + T_{i-\frac{1}{2},j}^{n} \cdot DFI_{i-1,j}^{\nu+1} + T_{i,j--\frac{1}{2}}^{n} \cdot DFI_{i,j-1}^{\nu+1} + +T_{i+\frac{1}{2},j}^{n} \cdot DFI_{i+1,j}^{\nu} + T_{i,j+\frac{1}{2}}^{n} \cdot DFI_{i,j+1}^{\nu}\right) \cdot SINV_{i,j}^{\nu} \quad (47)$$

where:

$SINV_{i,j}^{n}$ – is the inverse of the matrix $S_{i,j}^{n}$.

4. Case study

A reservoir with a length of 180 m, a width of 10 m and a thickness of 10 m was analyzed, over which a discretization grid with the step Δx = 10 m and Δy = 10 m (figure 5).

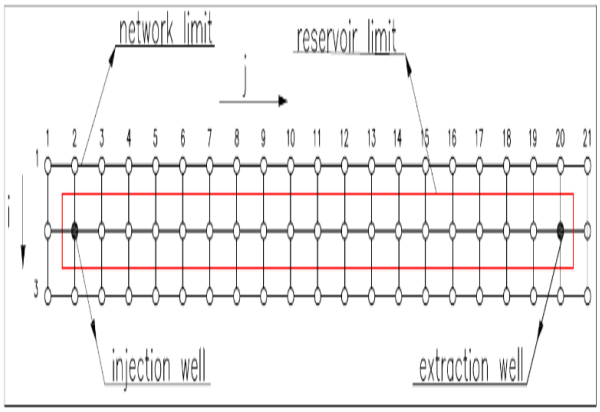

Figure 5 – The discretization network associated with the core reservoir

At one end of the reservoir is a well with flow rate Q_{inj} = 10 m^3/day, and at the other end an extraction well that flows 10 m^3/day of liquid (crude oil until the dislocation front penetrates, and then flows crude oil and water) into the purpose of maintaining the balance: the injected fluid flow is equal to the extracted fluid flow [9].

The reservoir sit has porosity m = 0.2 and permeability K = 0.5 D (Darcy). Also, the reservoir is initially uniformly saturated with saturation water S_{ai} = 0.2 and crude oil S_{t0} = 0.8.

The initial reservoir pressure is 100 at in the crude oil phase.

Considering that the classical Buckley-Leverett theory [2] doesn't consider the capillary pressure to make the comparison of the numerical solution with the analytical solution, an apparent variation of the capillary pressure with the normalized saturation S having a linear variation and with small values for P_c (the pressure capillary) (*figures 6*).

The properties of the fluid medium ensure the incompressibility of the system as in the Buckley-Leverett theory so that it is considered:

$$b_a = 1,\ b_t = 1,\ \mu_a = 1\ cP,\ \mu_t = 1\ cP,$$
$$\rho_a = 1\ kg/m^3,\ \rho_t = 1\ kg/m^3$$

The reservoir analyzed is horizontal, so $z_{i,j} = 0$ for any node (i,j).

The irreducible saturation in the crude oil phase $S_{tir} = 0$ is also considered [10].

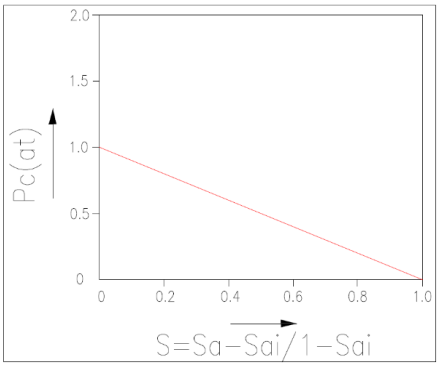

Figure 6 – The apparent variation of capillary pressure (P_c) with normalized saturation (S)

The relative permeabilities are those resulting from the simulation (figure 7) which can be expressed by the relations below:

$$k_a = \left(\frac{S_a - S_{ai}}{1 - S_{ai}}\right)^3 \quad (48)$$

$$k_t = \left(\frac{1 - S_a - S_{ai}}{1 - S_{ai} - S_{t_{ir}}}\right)^3 \quad (49)$$

According to the Buckley-Leverett theory [2], [4], the variation of the water fraction in the stream fa with S_a saturation and the df_a/dS_a variation with water saturation were simulated (figure 8), which leads to the identification of the saturation profile along the core at different times of calculation, in other words to finding out the variation $S_a\ (x,t)$ according to the relationship below:

$$x\,\Big|S_a = \frac{Q_{inj}}{m \cdot K \cdot \Delta y} \cdot \frac{df_a}{df_a}\Big|S_a \cdot t \quad (50)$$

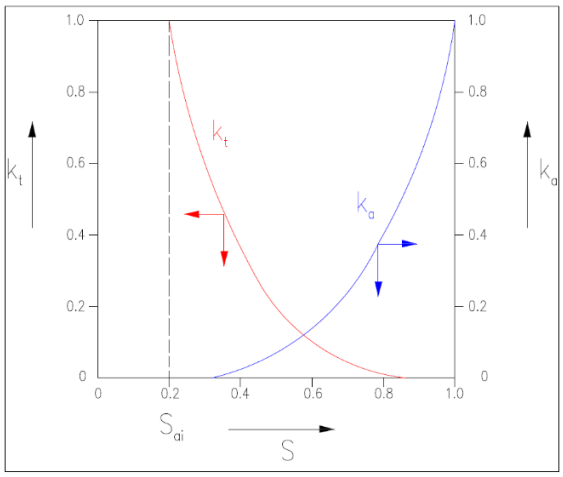

Figure 7 – The relative phase permeability curves

The research was carried out in two variants, one for the case of the SS Method and another for the case of the SSEC Method.
With these data using first the SS Method in the classical version and then the equations of the Gauss-Seidel Method for the calculation of DFIA and DFIT from each nodal point, a numerical solution was obtained which for t = 130 days shows, compared to the analytical solution, as is shown in the simulations in (figure 9).
Since the system equations (47) were treated separately, the DFIA and DFIT iterations were solved. In other words, the errors propagate differently on these unknowns.

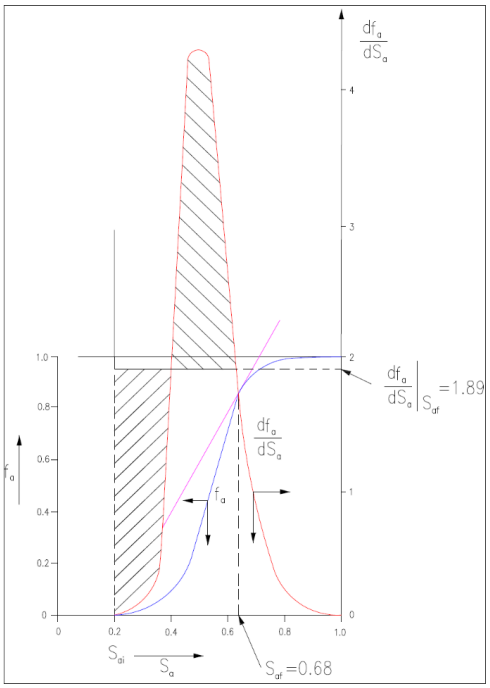

Figure 8 – The variation simulation f_a and df_a/S_a with S_a according to the theory Buckley-Leverett

The value of saturation in water is a function of the difference between DFIA and DFIT being thus strongly affected by the accuracy of the solution.
Given the fact that the numerical solution is calculated with a certain order of error (in the researched case a relative error of 1 ‰ is allowed for a number of 40 iterations), the results were strongly affected (according to figure 9) by the errors due to the method of calculation.
It should be noted that the system of equations is non-linear, the use of a large number of iterations to increase precision does not lead to realistic solutions.

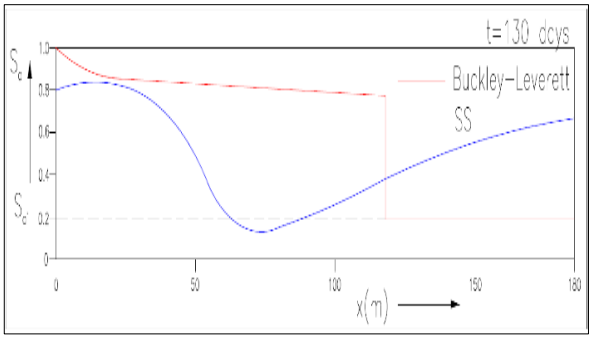

Figure 9 – The simulation of numerical solution of the SS Method compared to the Buckley-Leverett analytical solution

But applying the SSEC Method used to calculate the DFIA and DFIT displacements, the relationships in the Gaiss-Seidel variant, the results for the same number of iterations and the same imposed precision appear in good agreement with the analytical solution as they are simulated for a few calculation moments (*figures 10*).

By using a curve $P_c = P_c\ (S_a)$, even with a smaller range of variation, the profile of the dislocation front would have been even closer to the vertical, respectively to the theoretical conditions imposed by the analytical solution.

The representation of the variation of impurities in the extraction well shown in (figure 11) looks much closer to reality than the same variation but given by the Buckley-Leverett Method suggests.

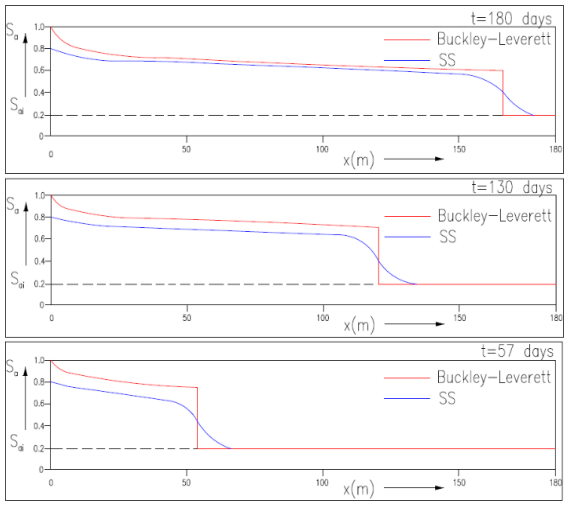

Figure 10 – The simulation of numerical solution of the SSEC Method compared to the Buckley-Leverett analytical solution

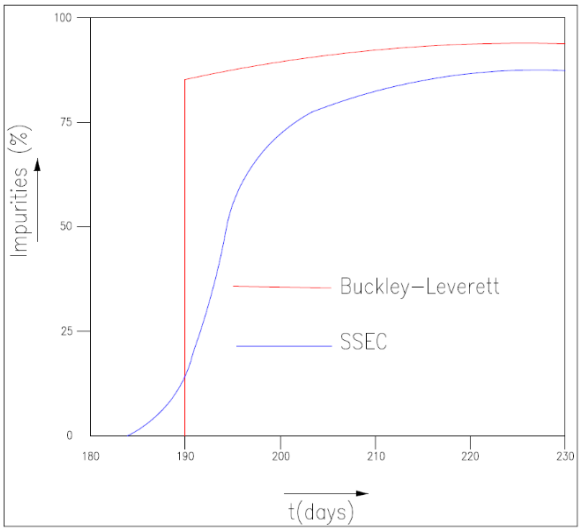

Figure 11 – The variation over time of the percentage of impurities from the production of the extraction well

5. Conclusions and proposals

The following conclusions are obtained from this research:

1. The Bouguer anomaly provided the alignments of the major gradient lines (G_1-G_8 and g_1-g_4) that allowed important conclusions to be drawn regarding the structure of the crystalline basement.
2. The regional anomaly analyzed through the three simulations in this research, reproduces the general morphology of the foredeep and specifies the transversal uplift sectors of the platform foundation and also provides information on the geological structures.
3. The local anomaly through the successive analysis of all the maximum and minimum terms from the three simulations carried out relative to the reference depths *l=1,2, 4 km*, respectively, bring an input of information on the depth at which the largest volume of sediments is located light (intermediate level indicated by the network with side *l=2 km*), the disturbing sources and the orientations of the local terms (the statistics of the directions of the major axes of the local terms show that 50 % of them have SW-NE orientations, 28% have W-E orientations and 11 % each they have N-S and NW-SE orientations, outcrops on the surface and in depth of the maximum and minimum zones corresponding to the major structures of the sedimentary deposits).
4. The geological elements that emerge from the interpretation of the elements of the gravity anomaly in the Subcarpathians of Muntenia, refer to:

- the structure of the crystalline basement;
- the structure of sedimentary deposits;

ISSN: 0067-3064

- causal relationships between the structure of sedimentary deposits and the structure of the crystalline foundation. Both categories of elements are synthesized in a simulation-gravimetric-geological structural map (*figure 4*).

5. Information regarding the structure of the crystalline foundation is grouped around:
- the nature of the crystalline basement (the magnetically differentiated foredeep foundation belonging to the Baikalian Platform composed of crystalline shales belonging to the Constanța-Cernavodă-Șăndărei sector);
- the compartmentalization and placement of the foundation blocks in steps (the flanks of the northern carpathian and southern platform foredeep are fractured and arranged each in four steps with opposite dips I-IV and IX-VI, axially the step of maximum immersion V with a complex structure).

6. Information about the structure of sedimentary deposits refers to:
- the nature and characterization of sedimentary deposits (a synthesis of characteristic facies and their distribution areas, density contrast surfaces);
- the area of maximum thickening of the sediments (corresponds to the area of maximum immersion of the crystalline basement);
- the major structures of the sedimentary deposits (distribution of volumes in mass contract with the appearance of successions of dense and light formations and with orientations and lengths significantly similar to the major gradient lines (G_1-G_8);
- the maximum and minimum areas correspond to about 14 major structures or ensembles of structures, present in the substratum of the researched region up to great depths, which follow each other symmetrically in relation to the steps of the crystalline basement.

7. Causal relationships between the structure of the sedimentary deposits and the structure of the crystalline basement are established by means of correlation elements (separated by category from the Bouguer anomaly, the regional anomaly and the local anomaly) and refer to:
- the distribution in the substrate at the level of the sedimentary deposits of dense and light masses (zoned arrangement of the major structures of the sedimentary deposits, the unitary or fragmented character of the major structures in relation to the steps and transversal elevations of the crystalline basement).

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